Exercise 27

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$g(x) = \sqrt{9 - x}$$

Solution

The domain of $g(x)$ is

$$9 - x \geq 0$$

$$-x \geq -9$$

$$x \leq 9$$

$$\{x \mid x \leq 9\}.$$

Calculate the derivative of $g(x)$ using the definition.

$$\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{9 - (x+h)} - \sqrt{9 - x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{9 - x - h} - \sqrt{9 - x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{9 - x - h} - \sqrt{9 - x}}{h} \cdot \frac{\sqrt{9 - x - h} + \sqrt{9 - x}}{\sqrt{9 - x - h} + \sqrt{9 - x}} \\
&= \lim_{h\to 0} \frac{(9 - x - h) - (9 - x)}{h\left(\sqrt{9 - x - h} + \sqrt{9 - x}\right)} \\
&= \lim_{h\to 0} \frac{-h}{h\left(\sqrt{9 - x - h} + \sqrt{9 - x}\right)} \\
&= \lim_{h\to 0} \frac{-1}{\left(\sqrt{9 - x - h} + \sqrt{9 - x}\right)} \\
&= \frac{-1}{\left(\sqrt{9 - x} + \sqrt{9 - x}\right)} \\
&= -\frac{1}{2\sqrt{9 - x}}
\end{aligned}$$

The domain of $g'(x)$ is

$$9 - x \geq 0 \quad \text{and} \quad 9 - x \neq 0$$

$$9 - x > 0$$

$$-x > -9$$

$$x < 9$$

$$\{x \mid x < 9\}.$$